

On Black Hole Structures in Scalar-Tensor Theories of Gravity

Kirill A. Bronnikov^{*1,3,4}, Júlio C. Fabris^{†2,3}, and Denis C. Rodrigues^{‡2}

¹VNIIMS, Ozyornaya ul. 46, Moscow 119361, Russia

²Departamento de Física - Universidade Federal do Espírito Santo - Brazil

³National Research Nuclear University “MEPhI,” Kashirskoe sh. 31, Moscow 115409, Russia

⁴ Institute of Gravitation and Cosmology, PFUR, ul. Miklukho-Maklaya 6, Moscow 117198, Russia

March 14, 2016

Abstract

We review some properties of black hole structures appearing in gravity with a massless scalar field, with both minimal and nonminimal coupling. The main properties of the resulting cold black holes are described. The study of black holes in scalar-gravity systems is extended to k -essence theories, and some examples are explicitly worked out. In these cases, even while the existence of horizons is possible, the metric regularity requirement on the horizon implies either a cold black type structure or a singular behavior of the scalar field.

^{*}kb20@yandex.ru

[†]fabris@pq.cnpq.br

[‡]deniscr@gmail.com

1 Black holes in general relativity

The existence of black holes is one of the most important predictions of the General Relativity theory. Even if some aspects of black hole physics can be obtained from the Newtonian theory, the complete concept of black holes includes the notion of an event horizon and a causal structure typical of the relativistic theory. We are perhaps close to a direct detection of black holes, and the amount of evidence on black holes is accumulating. The recent detection of gravitational waves by the LIGO experiment [1] is considered as a clear evidence for the existence of these objects since the measured gravitational wave carries the signature of the coalescence of two very massive black holes. Moreover, the centers of galaxies may be inhabited by black holes with masses of the order of millions to billions of solar masses. There are also many indications of the existence of black holes resulting from stellar evolution, with masses of the order of a few solar masses.

There are four most well-known black hole solutions in General Relativity: the uncharged static black hole, given by the Schwarzschild solution; the charged static black hole, the Reissner-Nordström solution; the uncharged rotating black hole, the Kerr solution; and the rotating, charged black hole, the Kerr-Newman solution. The properties of these solutions led to the so-called *no-hair conjecture*: in general, black holes must be characterised by only three parameters in Einstein-Maxwell theory, the mass M , the charge Q and the angular momentum J ; however, other parameters corresponding to new charges are added if more gauge fields are included into consideration, as happens, e.g., with string theory inspired Lagrangians; also new parameters like magnetic monopole charges may appear when general new gauge fields are taken into account. There is strong evidence that this conjecture may be violated in some situations, in particular, when new sources like scalar fields are considered — see the contributions of Herdeiro et al. in the present volume.

The prototype of the black hole structure is given by the Schwarzschild solution, for which the line element reads

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2. \quad (1)$$

Here, the crucial concept of an *event horizon* appears, through the coordinate singularity at $r_H = 2GM$. The Reissner-Nordström solution is given by a line element with two parameters, mass and charge, which reads

$$ds^2 = \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2. \quad (2)$$

It brings to scene the notion of a Cauchy horizon, here given by $r_- = M - \sqrt{M^2 - Q^2}$, while the event horizon appears at $r_+ = M + \sqrt{M^2 - Q^2}$. The Reissner-Nordström solution also implies the notion of an extremal black hole, represented by the configuration where the Cauchy horizon coincides with the

event horizon, $r_+ = r_-$, so that $M = Q$. Moreover, if $Q > M$, the horizons disappear, and the central singularity becomes accessible to an external observer: this is a naked singularity. These new features appear also, in a different form, in the Kerr and Kerr-Newman solutions.

One of the important results in the study of black hole physics is the discovery of the thermodynamics behavior of those objects. The temperature and entropy of the Schwarzschild black hole are given by

$$T_H = \frac{1}{8\pi} \frac{1}{M}, \quad S_H = 4\pi M^2 = \frac{A_H}{4}, \quad (3)$$

where A_H is the area of the event horizon surface. The temperature of the Reissner-Nordström black hole is given by

$$T_{RN} = \frac{1}{8\pi M} \left(1 - 16\pi^2 \frac{Q^4}{A^2} \right), \quad (4)$$

$$A = 4\pi R_+^2. \quad (5)$$

The temperature of the Reissner-Nordström black hole becomes zero when the extremal condition is satisfied, $M = Q$. There is no well-defined temperature for naked singularities. The entropy of the Reissner-Nordström black holes is still given by the area law:

$$S = \pi r_+^2. \quad (6)$$

The entropy is always finite, including the extremal case (see the contribution by Lemos in the present volume). But, it becomes ill-defined in the naked singularity case. For a general view of these standard results, see Refs. [2, 3]

This quick review of the standard results in standard black hole physics settles a basic framework to the main subject of the present text: the fate of static black hole solutions when the scalar field are added to the gravitational Lagrangian, with minimal or non-minimal coupling. Seminal studies in this sense have been presented in [4–6]. In some situations, new features appear: in general, black hole solutions in the presence of a scalar field require a violation of the standard energy conditions, so that the scalar field must have a phantom nature; the event horizon surface has an infinite area; the Hawking temperature is always zero. Essentially, these results have been obtained in Refs. [7, 8]. We review these results and present some new ones, with a similar structure, obtained in the context of k-essence models [9].

2 Scalar fields

The introduction of scalar fields is the simplest extension of General Relativity. It adds a spin zero degree of freedom to the theory. We can think on some different possibility to introduce a scalar field in a gravitational context. The first one makes use of the minimal coupling between the scalar field and gravitation,

and it is defined by the Lagrangian

$$\mathcal{L} = \sqrt{-g} \left\{ R - \epsilon \phi_{;\rho} \phi^{;\rho} + 2V(\phi) \right\}. \quad (7)$$

In this Lagrangian, $V(\phi)$ is the self-interaction potential. This kind of theory in general satisfies the energy conditions if $\epsilon = 1$ (an ordinary field) and violates the energy conditions if $\epsilon = -1$ (a phantom fields).

We may have a much less trivial configuration using a non-minimal coupling,

$$\mathcal{L} = \sqrt{-g} \left\{ f(\phi) R - \omega(\phi) \frac{\phi_{;\rho} \phi^{;\rho}}{\phi} + 2V(\phi) \right\}. \quad (8)$$

where $f(\phi)$ and $\omega(\phi)$ are, in general, functions of the scalar field. The case

$$f(\phi) = \phi, \quad \omega(\phi) = \text{constant}, \quad V(\phi) = 0, \quad (9)$$

defines the traditional Brans-Dicke theory, the prototype of the scalar-tensor theories.

Another possibility of including a scalar field is a non-canonical kinetic term, leading to k -essence models,

$$\mathcal{L} = \sqrt{-g} \left\{ R - \omega(\phi) f(X) + 2V(\phi) \right\}, \quad (10)$$

where

$$X = \phi_{;\rho} \phi^{;\rho}, \quad (11)$$

and f is a given function of X .

Other possibilities are the Horndeski theory [10], with the most general scalar-tensor Lagrangian leading to second-order differential equations, and the Galileon theory [11], where a scalar field exhibits some special translational symmetry. One example of such a generalization is the theory defined by

$$\mathcal{L} = \sqrt{-g} \left\{ R - G^{\mu\nu} \phi_{;\mu} \phi_{;\nu} + 2V(\phi) \right\}, \quad (12)$$

where $G_{\mu\nu}$ is the Einstein tensor.

3 Black holes in minimally coupled models

In the ordinary case where the scalar field is minimally coupled to gravity, without a potential, represented by the equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \phi_{;\mu} \phi_{;\nu} - \frac{1}{2} g_{\mu\nu} \phi_{;\rho} \phi^{;\rho}, \quad (13)$$

$$\square \phi = 0, \quad (14)$$

there is no black hole solution. Indeed, let us write the four-dimensional spherically symmetric, static metric as

$$ds^2 = e^{2\gamma} dt^2 - e^{2\alpha} du^2 - e^{2\beta} d\Omega^2, \quad (15)$$

where α , β and γ are functions only of the radial coordinate u . The field equations are

$$-2\beta'' - 3\beta'^2 + 2\beta'\alpha' + e^{2(\alpha-\beta)} = \frac{\epsilon}{2}\phi'^2, \quad (16)$$

$$2\gamma'\beta' + \beta'^2 - e^{2(\alpha-\beta)} = \frac{\epsilon}{2}\phi'^2, \quad (17)$$

$$\beta'' + \gamma'' + \beta'(\gamma' + \beta' - \alpha') + \gamma'^2 - \alpha'\gamma' = -\frac{\epsilon}{2}\phi^2, \quad (18)$$

$$\phi'' + [\gamma' + 2\beta' - \alpha']\phi' = 0. \quad (19)$$

For an ordinary scalar field, $\epsilon = 1$. Choosing the coordinate condition

$$\alpha = \gamma + 2\beta, \quad (20)$$

the scalar field equation can be easily solved:

$$\phi = \phi_0 u + \phi_1. \quad (21)$$

The remaining equations simplify to

$$-2\beta'' + \beta'^2 + 2\beta'\gamma' + e^{2(\gamma+\beta)} = \frac{\epsilon}{2}\phi'^2, \quad (22)$$

$$2\gamma'\beta' + \beta'^2 - e^{2(\gamma+\beta)} = \frac{\epsilon}{2}\phi'^2, \quad (23)$$

$$\beta'' + \gamma'' + \beta'^2 - 2\beta'\gamma' = -\frac{\epsilon}{2}\phi^2. \quad (24)$$

A combination of those equations leads to

$$\gamma'' = 0. \quad (25)$$

The solution is:

$$\gamma = bu. \quad (26)$$

Identifying,

$$e^{-2ku} = 1 - 2\frac{k}{x}, \quad (27)$$

the final metric is

$$ds^2 = P^a dt^2 - P^{-a} dx^2 - P^{1-a} x^2 d\Omega^2, \quad (28)$$

with

$$P(x) = 1 - 2\frac{k}{x}, \quad \phi = -\frac{C}{2k} \ln P(x), \quad (29)$$

$$\frac{m}{k} = a, \quad a^2 = 1 - \frac{C^2}{2k^2}. \quad (30)$$

In the expressions above, m is the ADM mass, C is a scalar charge in the sense that $\phi \approx C/x$ at large x (but with no universal conservation law in the present case) and k is their combination according to (30). The metric (28), together with the definitions (29,30), corresponds to Fisher's solution [4]. Clearly, there is a naked singularity at $x = 2k$: it is a center.

The situation changes drastically if the null energy condition is violated [7]. The null energy condition requires that

$$\rho + p \geq 0. \quad (31)$$

In order to violate this condition, the kinetic term must appear with the *wrong* sign. If $\epsilon = -1$ (a phantom field), we have

$$ds^2 = P^a dt^2 - P^{-a} dx^2 - P^{1-a} x^2 d\Omega^2, \quad (32)$$

$$a^2 = 1 + \frac{C^2}{2k^2} > 1. \quad (33)$$

This case represents a black hole with an event horizon at $x = 2k$ with two remarkable features: an infinite event horizon area and zero surface gravity.

4 Nonminimal coupling

We can write a general action with a non-minimal coupling without a potential as

$$\mathcal{L} = \sqrt{-g} \left\{ f(\phi) R - \omega(\phi) \frac{\phi_{; \rho} \phi^{; \rho}}{\phi} \right\}. \quad (34)$$

It is not, however, the most general (the Horndeski theory is the most general one leading to second-order equations), but it covers a large sample of theories. Let us fix

$$f(\phi) = \phi. \quad (35)$$

Performing a conformal transformation

$$g_{\mu\nu} = \phi^{-1} \tilde{g}_{\mu\nu}, \quad (36)$$

and writing

$$\frac{d\phi}{d\sigma} = \left| \frac{3 + 2\omega}{\phi^2} \right|^{1/2}, \quad (37)$$

we end up with

$$\mathcal{L} = \sqrt{-g} \left\{ R - \epsilon \sigma_{; \rho} \sigma^{; \rho} \right\}, \quad \epsilon = \pm 1. \quad (38)$$

A class of static, spherically symmetric solutions is given by

$$ds^2 = P^{-\xi} \left\{ P^a dt^2 - P^{-a} dx^2 - P^{1-a} x^2 d\Omega^2 \right\}, \quad (39)$$

with

$$P(x) = 1 - 2k/x, \quad \phi = P^\xi, \quad (40)$$

$$b/k = a, \quad a^2 = 1 - (3 + 2\omega)\xi^2, \quad (41)$$

$$2k^2 \text{sign} k = 2b^2 + \epsilon C^2. \quad (42)$$

Black holes are only possible when $3 + 2\omega < 0$, corresponding to the phantom configuration in the Einstein's frame.

Are the solutions described above, with a minimal or non-minimal coupling, black holes? They have some striking features:

1. Their surface gravity is zero. It means that their Hawking temperature (if it is possible to define it!) is zero. For this reason, these black holes has been called **cold black holes**.
2. The area of the event horizon is infinite. This means an infinite entropy if we follow the area law. But if the temperature is zero, the entropy cannot be in principle infinite. Hence, the computation of the entropy in this case may not follow the classical lines fixed for black holes emerging in General Relativity.
3. The tidal forces are infinite at the event horizon hypersurface. But point particles can safely cross this hypersurface.
4. The solutions are asymptotically flat. Hence, static observers at infinity can be defined.

There is another important feature: The solutions contain a scalar charge, hence these solutions can be called hairy black holes. This happens even in the Einstein frame if the energy conditions are violated. This fact points at a limitations of the so-called *no-hair theorem*, which seems to be restricted to very specific situations.

In fact, the general solution for these scalar black holes reveals two types of asymptotically flat black hole, besides the trivial (Schwarzschild) one [8]: Type B1 black holes, where the horizon can be crossed in a finite proper time by an infalling particle; Type B2 black hole, where the horizon is reached in a infinite proper time for an infalling particle. In both cases, there is always a throat on the way from spatial infinity to the horizon.

5 Stability

The usual GR black hole solutions are stable [2]. These Cold Black Holes, are they stable? The stability of black holes is a very delicate subject, technically

and conceptually. Technically because it is almost impossible to integrate the perturbed equations even in the simplest case, that of pure radial perturbations; conceptually because the results may be different for different perturbation formalisms.

Let us consider just radial perturbations:

$$\alpha = \alpha_0(u) + \delta\alpha(u, t), \quad \beta = \beta_0(u) + \delta\beta(u, t), \quad (43)$$

$$\gamma = \gamma_0(u) + \delta\gamma(u, t), \quad (44)$$

where α_0 , β_0 and γ_0 are the previously found background solutions and $\delta\alpha$, $\delta\beta$ and $\delta\gamma$ are (time-dependent!) perturbations around them. In analyzing perturbations around the Schwarzschild solution, for example, it is very convenient to choose the coordinate condition $\delta\beta = 0$. However, here the situation is more delicate since there is a throat in the solutions outside the horizon, the condition above turns out to be a *physical condition*: only perturbations that do not affect the throat are considered.

It is possible, for Cold Black Holes, to choose a coordinate condition similar to that of the background.

$$\delta\alpha = \delta\gamma + 2\delta\beta. \quad (45)$$

The perturbed equations then may be reduced to a single master equation connected with the perturbation of the scalar field:

$$e^{4\beta(u)}\delta\ddot{\phi} - \delta\phi'' = 0. \quad (46)$$

The primes mean derivative with respect to u . This equation can be reduced to

$$\delta\phi'' + e^{4\beta(u)}\omega^2\delta\phi = 0. \quad (47)$$

The condition of stability is fixed as follows: specify the boundary conditions at the horizon and at infinity; verify if there are solutions connecting these boundary conditions with $\omega^2 < 0$; if there are solutions satisfying the above conditions at both ends, we conclude that there are divergent growing modes, and the background static solution is unstable, if not, the background solution is stable. Following this procedure, we have found the following results [8]: the cold black hole solutions are stable; solutions with a naked singularity are unstable.

Let us consider now a scalar field and metric perturbations that transform under the radial reparametrisation Δu as

$$\delta\phi \rightarrow \bar{\delta}\phi = \delta\phi + \phi'\Delta u, \quad (48)$$

$$\delta r \rightarrow \bar{\delta}r = \delta r + r'\Delta u, \quad (49)$$

where we have defined $r(u) = e^{\beta(u)}$. The combination

$$\Psi = r'\delta\phi - \phi'\delta r, \quad (50)$$

is invariant under that coordinate transformation. The final equation is [12]

$$e^{2(\alpha-\gamma)}\delta\ddot{\phi} - \delta\phi'' - \delta\phi'(\gamma' + 2\beta' - \alpha') + U\delta\phi = 0, \quad (51)$$

$$U \equiv e^{2\alpha} \left\{ \epsilon(V - e^{-2\beta}) \frac{\phi'^2}{\beta'^2} + 2 \frac{\phi'}{\beta'} V_\phi + \epsilon V_{\phi\phi} \right\}, \quad (52)$$

where V is a possible potential according to the Lagrangian (7). The singularity due to the term with β' in the denominator prevents a consideration of solutions finite at the throat a throat. This singularity may be removed by a further transformation [13], and a Schrödinger-type equation with a regular potential is obtained. As a result of this analysis, all previous solutions which contain throats turn out to be unstable, including those corresponding to black holes [12]. The reason for this discrepancy is not clear. Perhaps, in the gauge $\delta\alpha = 2\delta\beta + \delta\gamma$, we end up with a master equation for a pure gauge mode. Or perhaps the singularity existing in the potential of the gauge-invariant approach has led to nonphysical modes.

6 Black holes in k -essence theories

Let us consider the Lagrangian of a k -essence model:

$$\mathcal{L} = \sqrt{-g} \left\{ R - f(X) - 2V(\phi) \right\}, \quad (53)$$

where $f(X)$ is a general function of the kinetic term for the scalar field,

$$X = \phi_{;\rho} \phi^{;\rho}. \quad (54)$$

For the static, spherically symmetric case, the equations of motion are:

$$-2\beta'' - 3\beta'^2 + 2\beta'\alpha' + e^{2(\alpha-\beta)} = e^{2\alpha} \left\{ \frac{1}{2}f(X) + V(\phi) \right\}, \quad (55)$$

$$2\gamma'\beta' + \beta'^2 - e^{2(\alpha-\beta)} = e^{2\alpha} \left\{ X f_X - \frac{1}{2}f(X) - V(\phi) \right\}, \quad (56)$$

$$2\gamma'\beta' + \beta'^2 - e^{2(\alpha-\beta)} = -e^{2\alpha} \left\{ \frac{1}{2}f(X) + V(\phi) \right\}, \quad (57)$$

$$\left[f_X e^{2\beta+\gamma-\alpha} \phi' \right]' = V_\phi. \quad (58)$$

For $V = 0$, it is possible to show [9] that it only cold black holes can exist, even in the absence of exact solutions. To do so, the coordinate condition is chosen to be

$$\alpha = -\gamma, \quad (59)$$

The following notations are used:

$$e^{2\gamma} = A, \quad e^\beta = r(u). \quad (60)$$

Different combinations of the equations lead to

$$2A \frac{r''}{r} = X f_X, \quad (61)$$

$$A(r^2)'' - A'' r^2 = 2, \quad (62)$$

$$2 \frac{r''}{r} = -C^2 \frac{f_X}{r^4}, \quad (63)$$

where C has the meaning of a scalar charge. A first integral can be obtained:

$$\left(\frac{A}{r^2} \right)' = \frac{6m - 2u}{r^4}. \quad (64)$$

A horizon implies

$$A \rightarrow 0. \quad (65)$$

But, there is the relation

$$A f_X \phi' = \frac{C}{r^2}. \quad (66)$$

Regularity at the horizon requires that the components of the energy-momentum tensor should be finite, and since they contain f_X and ϕ' , a singularity can only be avoided if

$$r \rightarrow \infty \quad (67)$$

Hence, a possible black hole must be cold.

Let us consider the particular case where

$$f(X) = X^n. \quad (68)$$

The equations of motion are the following:

$$-2\beta'' - 3\beta'^2 + 2\beta'\alpha' + e^{2(\alpha-\beta)} = \frac{\epsilon}{2} e^{2(1-n)\alpha} \phi'^{2n} + e^{2\alpha} V, \quad (69)$$

$$e^{-2\alpha} [2\gamma'\beta' + \beta'^2 - e^{-2\beta}] = (2n-1) \frac{\epsilon}{2} e^{-2n\alpha} \phi'^{2n} - V, \quad (70)$$

$$\beta'' + \gamma'' + \beta'(\gamma' + \beta' - \alpha') + \gamma'^2 - \alpha'\gamma' = -\frac{\epsilon}{2} e^{2(1-n)\alpha} \phi'^{2n} - e^{2\alpha} V, \quad (71)$$

$$(2n-1)\phi'' + [\gamma' + 2\beta' + (1-2n)\alpha']\phi' = \frac{\epsilon}{n} e^{-2(1-n)\alpha} \phi'^{2(1-n)} V_\phi. \quad (72)$$

A solution can be obtained for $n = 1/2$, $V = \Lambda = \text{constant}$ [9]: using the coordinate condition $\alpha = \gamma + 2\beta$:

$$ds^2 = \frac{\Lambda^2}{\cosh^4 \theta} dt^2 - du^2 - \frac{\cosh^2 \theta}{\Lambda^2} d\Omega^2, \quad (73)$$

$$\phi = \frac{2}{3} \epsilon \Lambda (-3\theta + 4 \tanh \theta) + \phi_0, \quad (74)$$

$$\theta = \frac{\Lambda}{3} u. \quad (75)$$

In the quasi-global coordinate system ($\alpha = -\gamma$), the metric has the form

$$ds^2 = (1 - \Lambda\rho^2)^2 dt^2 - \frac{d\rho^2}{(1 - \Lambda\rho^2)^2} - \frac{d\Omega^2}{(1 - \Lambda\rho^2)}. \quad (76)$$

This solution seems to represent a cold black hole. But it has a very curious structure. It is a kind of type B2 cold black hole. But, instead of having a flat asymptotic and a horizon, we have two horizons at $\rho = \pm 1/\sqrt{\Lambda}$, corresponding to $u \rightarrow \pm\infty$. Beyond each horizon, we find a timelike singularity at $\rho \rightarrow \pm\infty$.

In Ref. [9] we also found a solution for the particular case $n = 1/3$. This solution reads

$$ds^2 = \left(\frac{B_0}{k^2 x} - \frac{k^2}{2} x^3 \right) dt^2 - \left(\frac{B_0}{k^2 x} - \frac{k^2}{2} x^3 \right)^{-1} dx^2 - \frac{1}{k^2 x} d\Omega^2, \quad (77)$$

where k and B_0 are constants. This metric is completely regular at all positive x , including the horizon located at $x = k^{-1}(2B_0)^{1/4}$. However, the function $f(X)$ is singular at the horizon, in agreement with the no-go theorem proved above.

7 Conclusions

Scalar fields allow for obtaining exact black hole solutions, but only if the energy conditions are violated, otherwise only naked singularities appear. This statement is valid for both minimal and non-minimal couplings of the scalar field with gravity. (In the only known exception [6, 15] with a conformal scalar field, this field is infinite at the event horizon.) A striking feature of these black holes is that they have zero gravity and an infinite area of the event horizon. Contrary to the usual black holes, these cold black holes imply infinite tidal forces at the horizon. However, this is only true for massless fields with or without inclusion of a non-interacting electromagnetic field [14]. in full agreement with the existing no-hair theorems.

The situation changes if one includes other structures, like the string-inspired dilatonic interaction between scalar and electromagnetic fields [16–18]: in this case, a more conventional black-hole horizon can appear, with finite area and finite temperature, in the presence of a usual scalar field. However, if the scalar field is phantom, there emerge cold black hole solutions [17, 18]. There are also black hole solutions, albeit numerical, with "true" scalar hair (i.e. an independent scalar charge which is conserved) in Einstein's gravity minimally coupled to two real (or one complex) massive scalar fields, and thus obeying all energy conditions [22].

The stability issue looks somewhat controversial (see above), but most probably the gauge (44), in which scalar field perturbations decouple from the metric ones, excludes the physical unstable mode found in [12, 13].

k -essence models give origin to similar structures, but with some other exotic configurations, to be still better explored. In particular, we find, at least for

some particular cases, multiple horizons beyond which infalling observers hit singularities. In such cases the usual notion of a distant observer is inapplicable since there is no spatial infinity. The features of such solutions, including their stability, deserve more complete studies.

We here did not touch upon a broader area of possible black holes with scalar hair in cases where scalar fields possess nontrivial self-interaction potentials. In such cases, apart from general results like different no-hair and other no-go theorems (see, e.g., [19–22]), there are a large number of exact solutions of interest, including singular and regular black holes with normal and phantom scalar fields with and without electromagnetic fields, see, e.g., [23–26] and references therein.

Acknowledgments

We thank CNPq (Brazil) and FAPES (Brazil) for partial financial support. The work of KB was partly performed within the framework of the Center FRPP supported by MEPhI Academic Excellence Project (contract number 02.a03.21.0005, 27.08.2013).

References

- [1] LIGO Scientific Collaboration and Virgo Collaboration, *GW150914: First results from the search for binary black hole coalescence with Advanced LIGO*, arXiv:1602.03839 .
- [2] S. Chandrasekhar, *The mathematical theory of black holes* (Oxford university press, Oxford, 1992).
- [3] V.P. Frolov and A. Zelnikov, *Introduction to Black Hole Physics* (Oxford university press, Oxford, 2011).
- [4] I. Z. Fisher, *Zh. Eksp. Teor. Fiz.* **18** (1948) 636; gr-qc/9911008.
- [5] H. Ellis, *J. Math. Phys.* **14** (1973) 104.
- [6] K. A. Bronnikov, *Acta Phys. Pol.* **B4** (1973) 251.
- [7] K.A. Bronnikov, M.S. Chernakova, J.C. Fabris, N. Pinto-Neto and M.E. Rodrigues, *Int. J. Mod. Phys.* **D17** (2008) 25.
- [8] K.A. Bronnikov, G. Clement, C.P. Constantinidis and J.C. Fabris, *Grav. Cosmol.* **4**128 (1998) 128.
- [9] K.A. Bronnikov, J.C. Fabris and D.C. Rodrigues, *Grav. Cosmol.* **22** (2016) 26.
- [10] Sh. Tsujikawa, *Lect. Notes Phys.* **892** (2015) 97.

- [11] C. Deffayet and D. A Steer, *A formal introduction to Horndeski and Galileon theories and their generalizations*, arXiv:1307.2450.
- [12] K.A. Bronnikov, J.C. Fabris, A. Zhidenko, *Eur. Phys. J.* **C71** (2011) 1791.
- [13] J.A. Gonzalez, F.S. Guzman and O. Sarbach, *Class. Quant. Grav.* **26** (2009) 015010.
- [14] K.A. Bronnikov, C.P. Constantinidis, R. Evangelista and J.C. Fabris, *Int. J. Mod. Phys.* **D8** (1999) 481.
- [15] N.M. Bocharova, K.A. Bronnikov and V.N. Melnikov, *Vestn. MGU, Fiz., Astron.*, No. 6 (1970) 706.
- [16] K.A. Bronnikov and G.N. Shikin, *Russ. Phys. J.* **20** (1977) 1138.
- [17] G. Clement, J.C. Fabris and M.E. Rodrigues, *Phys. Rev.* **D79** (2009) 064021.
- [18] G. Clement, J.C. Fabris and G.T. Marques, *Phys. Lett.* **B651** (2007) 54.
- [19] S. Adler and R.B. Pearson, *Phys. Rev.* **D 18** (1978) 2798.
- [20] J.D. Bekenstein, in: *Cosmology and Gravitation*, ed. M. Novello (Atlantis-science, Biarritz, 2000), pp. 1-85.
- [21] K.A. Bronnikov, *Phys. Rev.* **D64** (2001) 064013.
- [22] C.A.R. Herdeiro and E. Radu, *Asymptotically flat black holes with scalar hair: a review* , arXiv:1504.08209.
- [23] K.A. Bronnikov and S.G. Rubin, *Black Holes, Cosmology, and Extra Dimensions*, World Scientific, Singapore(2012).
- [24] K.A. Bronnikov and J.C. Fabris, *Phys. Rev. Lett.* **96** (2006) 251101; gr-qc/0511109.
- [25] K.A. Bronnikov, V.N. Melnikov and H. Dehnen, *Gen. Rel. Grav.* **39** (2007) 973; gr-qc/0611022.
- [26] S.V. Bolokhov, K.A. Bronnikov, and M.V. Skvortsova, *Class. Quant. Grav.* **29** (2012) 245006; ArXiv: 1208.4619.